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On the Fuzzy Reliability Estimation for standard Kumaraswamey Distribution

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Abstract

In this article, we considered tow procedures to estimate the fuzzy reliability for Kumaraswamy distribution, first depends on fuzzy reliability definition that uses the trapezoidal rule in order to find the numerical integration, and second is Bayesian procedure which in clouds different cases depends on sample data and hyper –parameters of prior gamma distribution.

Keywords: Fuzzy reliability, Kumaraswamy distribution, Maximum Likelihood, Bayesian estimation, Membership functions.

1. Introduction

Kumaraswamy distribution is very similar to the beta distribution and has the same basic shape; it's often referred to as a "Beta-like" distribution. However, the Kumaraswamy distribution is simpler to use and in some situations more tractable. In reliability and life testing experiments, many times the data are modeled by Kumaraswamy distribution.

Kumaraswamy distribution (KUM) named by Kumaraswamy which introduced a distribution for double bounded random processes with hydrological applications and it is a continuous probability density defined on the interval (0.1) and is very similar to beta distribution. The distribution function was defined in (1980) and written as following:

If X is distributed as Kumaraswamy non-negative random variable with shape parameter p >0 and scale parameter q >0, then its cumulative distribution function (CDF) and probability density function (PDF) are respectively given by:

$$f_z(x) = \frac{1}{b-c}\, pq \left(\frac{x-c}{b-c}\right)^{p-1} \left[1 - \left(\frac{x-c}{b-c}\right)^{p}\right]^{a-1} \quad (1)$$

$$F(x) = 1 - (1 - x^p)^q \quad (2)$$

To extract the standard distribution formula when c = 0, b = 1 the function becomes as follow:

$$f_x(x) = pq\, x^{p-1}\, [1 - x^p]^{q-1} \quad (3)$$

$$F(x) = 1 - (1 - x^p)^q \quad (4)$$

Then the reliability function of the tow parameter Kumaraswamy random variable is given by:

$$R(x) = (1 - x^p)^q \quad (5)$$

2. Fuzziness and fuzzy sets:

A person faces daily decision-making in a specific situation, and the decision-making includes uncertainty resulting from the lack of information related to the decision-making. The fuzziness or (uncertainty, and confusion) exists wherever and when people interact with the real world, and who the types of uncertainty in the reliability of fuzzy systems are the following

- Imprecision as failure and maintenance times.
- Incomplete data, such as lack of data and control test data.
- Vagueness as in human error and linguistic description of performance characteristics such as (good, acceptable, not good, ... etc).
- Randomness as in vehicle failure and measurement error.
- Subjectivity as in expert opinion and lack of knowledge.
- Complexity, as in the relationships between system and compounds, and the interaction between subsystems.

3. Membership function:

The Membership function is one of the most important functions in the theory of fuzzy sets, which is used to represent various types of fuzzy sets. It is the function that produces values within the interval [0, 1] to express the degree of membership of each element in the universe set to the fuzzy set, in other words it is the map that draws the degree of validity (degree of membership achievement) for each element in the universe belonging to the fuzzy set, It is a positive value function, and the basic condition for this function is that its range is between zero and one, then,

$$\mu_{\tilde{A}}(x_i) = \{(x_i, \mu_{\tilde{A}}(x_i)), x \in X, i = 1,2,3, \dots \dots n,\ then,\ 0 < \mu_{\tilde{A}}(x) < 1\} \quad (6)$$

4. Fuzzy numbers:

used to describe the state of uncertainty, and they are numbers that are often triangular, trapezoidal, or any other shape.

The fuzzy number is a fuzzy set with the following conditions:

- Convex and Normalized
- The membership function $\mu_{\tilde{A}}(x_i)$ is semi-continuous from the top in a real numbers R
- The level set α is determine for each α ϵ [0,1]

The Fuzzy Number types as following:

i) Triangular membership function

It is defined by three numbers $\boldsymbol{a_1, a_2, a_3}$ where, $\boldsymbol{a_1 < a_2 < a_3}$ and the base of the triangle is the interval $[\boldsymbol{a_1, a_3}]$ and its head is at $\boldsymbol{x = a_2}$ and can be written in the following form:

$$\boldsymbol{\tilde{N} = (a_1/a_2/a_3)}$$

And the fuzzy triangular number $\boldsymbol{\tilde{N} = (a_1/a_2/a_3)}$ is distinguished by the triangular membership function and its formula is as follows:

$$\mu_{\tilde{N}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & o.w \end{cases} \quad (7)$$

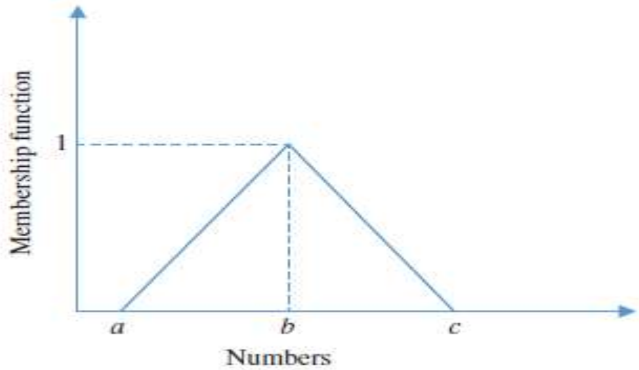

Fig. **(1)** Triangular membership functions

ii) Trapezoidal Fuzzy Number:

It is defined by four numbers: a_1, a_2, a_3, a_4, where $a_1 < a_2 < a_3 < a_4$ and the base of the triangle is the period $[a_1, a_4]$ and its top at the interval $[a_2, a_3]$and can be written in the following form:

$$\boldsymbol{\tilde{M} = (a_1/a_2, a_3/a_4)}$$

The trapezoidal fuzzy number $\tilde{M} = (a_1/a_2, a_3/a_4)$ is characteristic of the trapezoidal membership function and its formula is as follows:

$$\mu_{\tilde{M}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ 0 & o.w \end{cases} \quad (8)$$

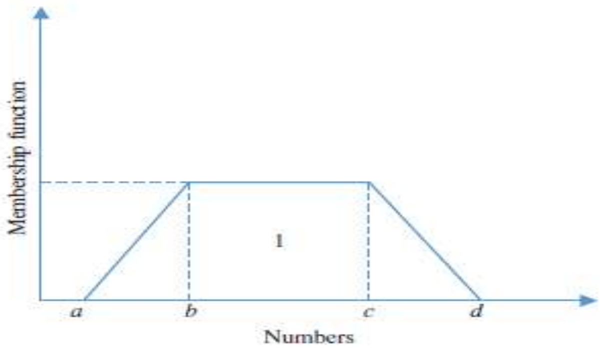

Fig. (2) Triangular membership functions

iii) Gaussian fuzzy number:

If the function of the membership function that almost has the shape of a bell, as functions of this type are a good alternative to the trigonometric functions that have an inflection point ((c ± b / 2) on each side of the function.

$$\mu_{\tilde{M}}(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\left(\frac{x-\mu}{2\sigma^2}\right)^2\right)} \quad (9)$$

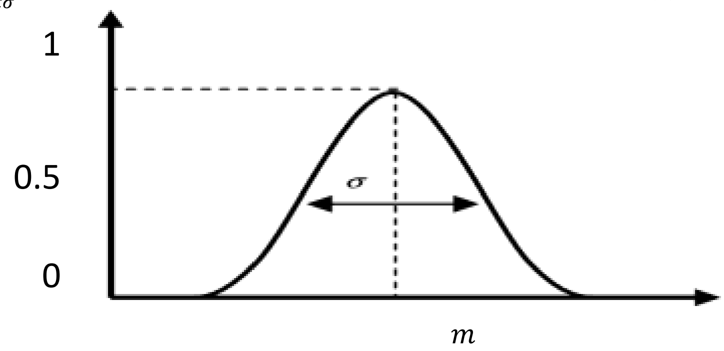

Fig. (3) The Gaussian Fuzzy number

5. Fuzzy sample space:

It is the fuzzy parts $\tilde{x} = (\tilde{x}_1, \dots\dots\dots, \tilde{x}_n)$ of $X = (X_1, \dots\dots\dots, X_n)$, in other words it is the set of fuzzy sub-sets of X with functions of membership is the Borel Measure, and verify orthogonally:

$$\sum_{\tilde{x}\in X} \mu_{\tilde{x}}(x) = 1$$

For each xϵ x, it is also called a fuzzy information system (FIS).

6. Fuzzy Reliability

Let T be the continuous random variable respectively the failure time of a system or unit or component then by using the formula of fuzzy probability. The fuzzy reliability can be obtained as following,

$$\tilde{R}(T) = p(T \geq t) = \int_{t_1}^{\infty} \mu(x) f(x)dx \ ; \ o \leq t \leq x < \infty \quad (10)$$

Were $\mu(x)$ is a the membership function represent for every element of a given universe, the degree to which this element belongs to fuzzy set:

Now ,assume that $\mu(x)$ is,

$$\mu(x) = \begin{cases} 0 & x \leq t_1 \\ \frac{x-t_1}{t_2-t_1} & t_1 \leq x \leq t_2 \ ; \ t_1 \geq 0 \\ 1 & x \geq t_2 \end{cases} \quad (11)$$

For $\mu(x)$ by computational method of the function of fuzzy number, the lifetime $x(\alpha)$ can be obtained corresponds to a certain value of α-cat, α ∈[0,1],as follows,

$\mu(x) = \alpha \ \rightarrow \frac{x-t_1}{t_2-t_1} = \alpha$, then

$$x(\alpha) \leq t_1 \ ; \ \alpha=0$$
$$x(\alpha) \leq t_1 + \alpha(t_2 - t_1) \ ; \ 0 < \alpha < 1 \quad (12)$$
$$x(\alpha) \geq t_2 \ ; \ \alpha=1$$

Thus ,the fuzzy reliability values can be obtained for all values of α as,

$$\tilde{R}(t)_{\alpha=0} = \int_{t_1}^{t_1} f(x)dx \quad (13)$$

$$\tilde{R}(t)_{0<\alpha<1} = \int_{t_1}^{x(\alpha)= t_1+\alpha(t_2-t_1)} f(x)dx \quad (14)$$

$$\tilde{R}(t)_{\alpha=1} = \int_{t_1}^{t_2} f(x)dx \quad (15)$$

7. Fuzzy Reliability of Kumaraswamy distribution:

we consider tow procedures to estimate the fuzzy reliability of a Kumaraswamy distribution, first procedure depends on the definition of fuzzy reliability and the second is Bayesian procedure .assume that $f(x)$) represent the pdf of Kumaraswamy distribution,then

First procedure: the fuzzy reliability definition,

$$\tilde{R}(t)_{\alpha=0} = \int_{t_1}^{x(\alpha)=0} \mathrm{pq} \ x^{\mathrm{p}-1} \ [1 - x^{\mathrm{p}}]^{\mathrm{q}-1} \ dx = 0 \quad (16)$$

$$\tilde{R}(t)_{0<\alpha<1} = \mathrm{pq} \int_{t_1}^{x(\alpha)(0<\alpha<1)} \frac{x-t_1}{t_2-t_1} x^{\mathrm{p}-1} \ [1 - x^{\mathrm{p}}]^{\mathrm{q}-1} dx \quad (17)$$

$$\tilde{R}(t)_{\alpha=1} = \mathrm{pq} \int_{t_1}^{t_2} x^{\mathrm{p}-1} \ [1 - x^{\mathrm{p}}]^{\mathrm{q}-1} dx \quad (18)$$

8. Fuzzy Maximum likelihood for KUM:

Let p,q represented by its maximum likelihood estimate that can be obtained from the likelihood function $L(q,p;\tilde{x})$.

$$L(q,p;\tilde{x}) = p(\tilde{x};q,p) = \prod_{i=1}^{n} f(\tilde{x};q,p)$$

$$L_0(q,p;\tilde{x}) = \prod_{i=1}^{n} \int_0^1 \mathrm{pq}\, x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}]^{\mathrm{q}-1} \mu_{\tilde{x}_i}(x)dx$$

$$L_0(q,p;\tilde{x}) = q^n p^n \prod_{i=1}^{n} \int_0^1 x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}]^{\mathrm{q}-1} \mu_{\tilde{x}_i}(x)dx \qquad (19)$$

By takeing the natural logarithm, $L_0(q,p;\tilde{x})$ will be ,

$$L^* = \log(L_0(q,p;\tilde{x})) = nlog(q) + n\log(p) + \sum_{i=1}^{n} \ln(\int_0^1 x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}]^{\mathrm{q}-1} \mu_{\tilde{x}_i}(x)dx) \qquad (20)$$

The estimates of the maximum Likelihood estimators for the parameters q and p can be obtained by maximizingL^*, partial differentiation for parameters q and p, and equating the result to zero as follows:

$$\frac{\partial L^*}{\partial q} = \frac{n}{\hat{q}} + \sum_{i=1}^{n} \frac{\int_0^1 [\, x^{\hat{\mathrm{p}}-1} [1 - x^{\hat{\mathrm{p}}}\,]^{\hat{\mathrm{q}}-1} \ln(1 - x^{\hat{\mathrm{p}}})] \mu_{\tilde{x}_i}(x)dx]}{\int_0^1 x^{\hat{\mathrm{p}}-1}\, [1 - x^{\hat{\mathrm{p}}}\,]^{\hat{\mathrm{q}}-1} \mu_{\tilde{x}_i}(x)dx} = 0 \qquad (21)$$

$$\frac{\partial L^*}{\partial p} = \frac{n}{\hat{p}} + \sum_{i=1}^{n} \frac{\int_0^1 [\, x^{\hat{\mathrm{p}}-1} (\hat{q}-1)\hat{p}x^{\hat{\mathrm{p}}-1} [1 - x^{\hat{\mathrm{p}}}\,]^{\hat{\mathrm{q}}-2} + [1 - x^{\hat{\mathrm{p}}}\,]^{\hat{\mathrm{q}}-1} (\mathrm{p}-1)x^{\hat{\mathrm{p}}-2}] \mu_{\tilde{x}_i}(x)dx]}{\int_0^1 x^{\hat{\mathrm{p}}-1}\, [1 - x^{\hat{\mathrm{p}}}\,]^{\hat{\mathrm{q}}-1} \mu_{\tilde{x}_i}(x)dx} = 0 \qquad (22)$$

9. Fuzzy Standard Bayes estimation for KUM:

The second procedure **:** according to the Bayesian procedure assume that p and q as a gamma prior distributing (p,q) with hyper –parameters a and b, then the posterior distribution

$$\pi_1(q) = \frac{d^c}{\Gamma c} q^{c-1} \exp(-dq) \qquad (23)$$

$$\pi_2(p) = \frac{b^a}{\Gamma a} p^{a-1} \exp(-bp) \qquad (24)$$

Based on those prior probability distributions (23), (24), the joint posterior density functions for q and p is written as follows:

$$\pi_1(q,p,\tilde{x}) = \frac{\pi_1(q).\pi_2(p).\ell(q,p;\tilde{x})}{\iint \pi_1(q).\pi_2(p).\ell(q,p;\tilde{x})dqdp} \qquad (25)$$

Now according to (13), (14) and (15), we get

$$\widetilde{R}\,(t)_{\alpha=0} \qquad (26)$$

$$\tilde{R}(t)_{0<\alpha<1} = \frac{\boldsymbol{pq^2}}{\boldsymbol{t_2 - t_1}} \left[(\boldsymbol{x(\alpha)^p}(\boldsymbol{x(\alpha)^p} - \boldsymbol{1}))^{\frac{\boldsymbol{q}}{\boldsymbol{p}}} - \left((\boldsymbol{t_1}^{\boldsymbol{p}}(\boldsymbol{t_1}^{\boldsymbol{p}} - \boldsymbol{1})) \right) \right] - \boldsymbol{t_1}[(\boldsymbol{1} - \boldsymbol{x(\alpha)^p}) - (\boldsymbol{1} - \boldsymbol{t_1}^{\boldsymbol{p}})]$$

(27)

$$\widetilde{R}(t)_{\alpha=1} = [1 - t_2{}^{\mathrm{p}}\,]^{\mathrm{q}} - [1 - t_1{}^{\mathrm{p}}\,]^{\mathrm{q}} \qquad (28)$$

Thus, the Bayesian estimator for the fuzzy reliability function of the KUM distribution is as follows:

$$\tilde{R}(t)_{0<\alpha<1} = \int_p \int_q \frac{pq^2}{t_2 - t_1} \left[(x(\alpha)^p(x(\alpha)^p - 1))^{\frac{q}{p}} \left((t_1{}^p(t_1{}^p - 1)) \right) \right] t_1[(1 - x(\alpha)^p)\,(1 - t_1{}^p)]\, \frac{q^{n+c-1}\, p^{n+a-1} \exp(-(dq+bc)) \prod_{i=1}^{n} \int_0^1 x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}\,]^{\mathrm{q}-1} dx}{\iint q^{n+c-1}\, p^{n+a-1} \exp(-(dq+bc)) \prod_{i=1}^{n} \int_0^1 x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}\,]^{\mathrm{q}-1} dx\; dqdp}\, \mathrm{dpdq} \qquad (29)$$

$$\tilde{R}(t)_{\alpha=1} = \int_p \int_q [1 - t_2{}^p\,]^q - [1 - t_1{}^p\,]^q \frac{q^{n+c-1}\, p^{n+a-1} \exp(-(dq+bc)) \prod_{i=1}^{n} \int_0^1 x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}\,]^{\mathrm{q}-1} dx}{\iint q^{n+c-1}\, p^{n+a-1} \exp(-(dq+bc)) \prod_{i=1}^{n} \int_0^1 x^{\mathrm{p}-1}\, [1 - x^{\mathrm{p}}\,]^{\mathrm{q}-1} dx\; dqdp}\, \mathrm{dpdq} \qquad (30)$$

For Bayesian procedure, with (29),(30), we consider the following three cases to estimate the fuzzy reliability with,

1. fuzzy observations ,xi = $\mu(x)$
2. Precise observations and fuzzy hyper-parameter b=b(α) with

$\mu(x) = \alpha \;\; \rightarrow \frac{x - t_1}{t_2 - t_1} = \alpha$, were b1 and b2 represents the lower and upper confidence interval of b ,and then we have ,

$$b(\alpha) \le b1 \qquad ; \quad b=0$$
$$b(\alpha) \le b_1 + \alpha(b_2 - b_1) \qquad ; \quad 0 < b < 1 \qquad (31)$$
$$b(\alpha) \ge b_2 \qquad ; \quad b=1$$

3-fuzzy observations ,xi= $\mu(x)$ and fuzzy hyper-parameter , b=b(α).

10-Applied data description:

The idea of taking samples about restorative ceramics for teeth was born because the cost of dental restoration is sometimes high, especially in advanced cases of dental diseases, as well as the difficulty of obtaining failure times that represent the failure of restorative ceramics to perform its function. They had records to record the time of building restorative ceramics for the teeth of the patients who visited the center. As every patient who visits the center for the purpose of restoring a tooth, a ceramic filling, or something about dental restoration, a file is opened for him inside the center, and this file remains even when the same patient is reviewed in the next times, so the time of the beginning of the restoration is recorded and until the patient is reviewed again when a fracture occurs Or damage or failure in the restoration, and the time of return is recorded. Thus, times were obtained representing the time until failure of the restorative dental ceramics for the auditors of the center in days during a period of 5 years, represented by (50) views. The estimation methods that were presented in the theoretical side will be applied for the purpose of calculating the reliability of restorative ceramics. For my dental agencies:

11-Data Fitting Test

For the purpose of knowing the real data distribution that represents the failure times of the cardiac stent, the (Easy Fit) program was used based on the tests **(Kolmogorov-Smirnov, Anderson-Darling, Chi-Squared)**. For testing the following hypotheses:

H_0: The data have the Kumaraswamy distribution

H_1: The data does not have the Kumaraswamy distribution

The results as shown in Table(3)

We note from Table (3) that the value of Sig. for all tests greater than the value of the level of significance of 0.05 and for the distribution, so we don't reject the null hypothesis.

12-Data Analysis:

The estimation methods used in the theoretical side were used to estimate the fuzzy reliability function for the Kumaraswamy distribution for the real data that represent the failure times of dental restoration ceramics as follows**:**

The table (4) showed the failure time of dental restoration ceramics and the degree of membership for each time, and the real fuzzy reliability, maximum likelihood estimation for fuzzy reliability, , and Bayes estimation for fuzzy reliability for Kumaraswamy distribution:

Table (1) shows the applied data.

t_i	0.14	0.16	0.19	0.24	0.29	0.54	0.80	1.47	3.23	9.43
t_i	0.07	0.16	0.27	0.43	0.44	0.83	1.81	2.11	2.97	4.27
t_i	0.08	0.21	0.25	0.38	0.41	0.47	1.26	5.82	7.90	11.24
t_i	0.06	0.15	0.28	0.28	0.32	0.49	0.66	1.48	1.69	2.06
t_i	0.10	0.11	0.18	0.23	0.25	0.65	1.51	1.65	2.89	9.38

Table (2) shows the values of the real sample statistics, as follows:

Table (2) Real Sample Statistics.

Statistic	Value
Mean	1.6458
Std. Error of Mean	.37265
Median	.4550
Mode	.16
Std. Deviation	2.63505
Variance	6.943
Range	.06
Minimum	11.24
Maximum	1.6458

Table (3) data fitting tests.

		Kolmogorov-Smirnov		**Anderson-Darling**		**Chi-Squared**	
		Statistic	**P-value**	**Statistic**	**P-value**	**Statistic**	**P-value**
Kumaraswamy	**Tests**	0.185577	0.05567	6.7711	0.1345	-	-
	Parameters estimates	**q**	0.39844		**p**	1.05335	

Table (4) fuzzy real reliability and fuzzy reliability under the methods of estimation for Kumaraswamy distribution.

t	Membership	R_Real	R_MLE	R_Bayes
0.14	0.007156	0.98926	0.99108	0.99925
0.16	0.008945	0.98541	0.98907	0.99885
0.19	0.011628	0.98098	0.98697	0.99835
0.24	0.016100	0.97045	0.98253	0.99697
0.29	0.020572	0.96435	0.98020	0.99608
0.54	0.042934	0.94290	0.97286	0.99245
0.8	0.066190	0.93473	0.97030	0.99090
1.47	0.126118	0.92608	0.96769	0.98916
3.23	0.283542	0.92608	0.96769	0.98916
9.43	0.838104	0.90737	0.96235	0.98510
0.07	0.000894	0.89736	0.95961	0.98276
0.16	0.008945	0.87608	0.95402	0.97742
0.27	0.018784	0.85325	0.94829	0.97116
0.43	0.033095	0.84131	0.94537	0.96768
0.44	0.033989	0.82903	0.94243	0.96395
0.83	0.068873	0.82903	0.94243	0.96395
1.81	0.156530	0.80356	0.93645	0.95574
2.11	0.183363	0.79041	0.93343	0.95125
2.97	0.260286	0.79041	0.93343	0.95125
4.27	0.376565	0.77701	0.93037	0.94649
0.08	0.001789	0.73550	0.92107	0.93063
0.21	0.013417	0.64843	0.90192	0.89156
0.25	0.016995	0.60393	0.89212	0.86835
0.38	0.028623	0.57424	0.88552	0.85155
0.41	0.031306	0.55945	0.88220	0.84276
0.47	0.036673	0.51546	0.87216	0.81487
1.26	0.107335	0.48661	0.86542	0.79508
5.82	0.515206	0.41695	0.84842	0.74174
7.9	0.701252	0.28153	0.81049	0.60964
11.24	1.000000	0.27068	0.80702	0.59697
0.06	0.000000	0.14661	0.75835	0.41856
0.15	0.00805	0.12660	0.74794	0.38184
0.28	0.019678	0.00854	0.60365	0.04982
0.28	0.019678	0.00153	0.53845	0.01043
0.32	0.023256	0.00140	0.53546	0.00958
0.49	0.038462	0.00107	0.52653	0.00738
0.66	0.053667	0.00028	0.48609	0.00193
1.48	0.127013	0.00019	0.47492	0.00127
1.69	0.145796	0.00005	0.44246	0.00032
2.06	0.178891	0.00000	0.37993	0.00001
0.1	0.003578	0.00000	0.36825	0.00000
0.11	0.004472	0.00000	0.21978	0.00000
0.18	0.010733	0.00000	0.20786	0.00000
0.23	0.015206	0.00000	0.17281	0.00000
0.25	0.016995	0.00000	0.07883	0.00000
0.65	0.052773	0.00000	0.02175	0.00000
1.51	0.129696	0.00000	0.00322	0.00000
1.65	0.142218	0.00000	0.00075	0.00000
2.89	0.253131	0.00000	0.00071	0.00000
9.38	0.833631	0.00000	0.00011	0.00000

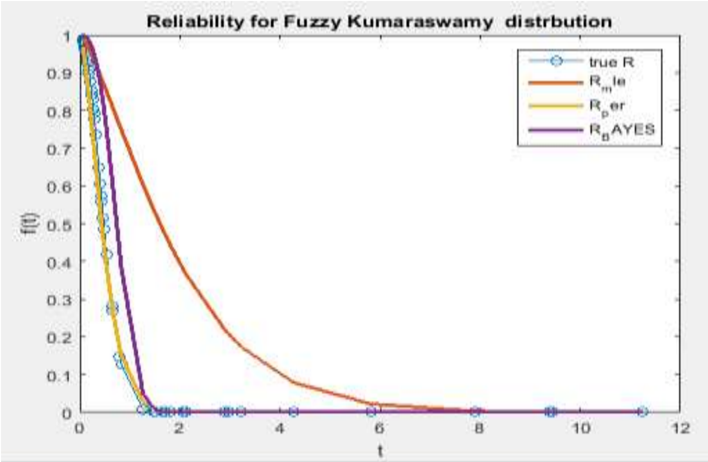

Fig. (4) curves of fuzzy reliability under the methods of estimation for Kumaraswamy distribution.

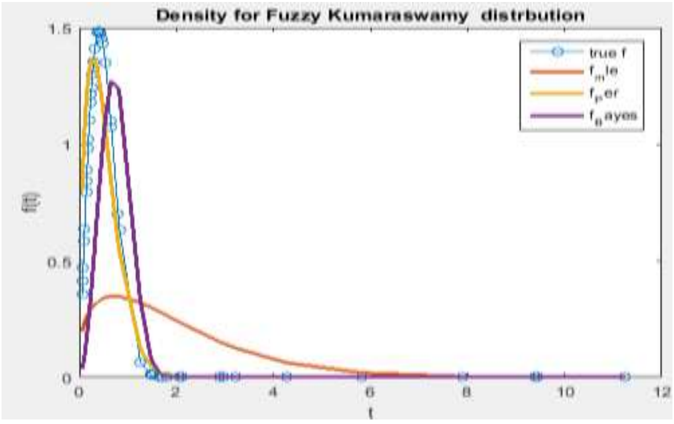

Fig. (5) curves of fuzzy density curve under the methods of estimation for Kumaraswamy distribution.

13. Results discussions

,we have discussed tow estimation procedures for the Kumaraswamy distribution Fuzzy reliability of the first procedure: depends on fuzzy reliability definition that uses the composite trapezoidal rule in order to find the numerical integration and the second: is Bayesian procedure with in formative gamma prior based on squared error and precautionary in Bayesian ,we have proposed to consider two different cases: 1.when the data are available in the form of fuzzy information "fuzzy observations". Precise observation with fuzzy hyper-parameter. Table (4) and Figures (4) and (5) show that the best way to estimate the fuzzy reliability function of the standard Kumaraswamy distribution is the Bayes method because it is the closest to the true fuzzy reliability values, followed by the standard maximum Likelihood method, , we note that the fuzzy reliability estimated according to the estimation methods is decreasing with time. The greater the time of failure of the restorative ceramics for the teeth, the less the reliability of the restorative ceramics for the teeth.

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